Numerical Simulation of Transient Diabatic Pipe Flow by using the Method of Characteristics

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Abstract

The following paper presents a one-dimensional numerical model for simulating transient thermohydraulic pipe flow based on the Method of Characteristics. In addition to mass and momentum conservation, the proposed scheme also guarantees compliance with the laws of thermodynamics by solving the energy equation. The model covers transient changes in fluid properties due to pressure changes, heat transfer and dissipation. The presented methodology also allows the computation of the transient temperature distribution in the pipe wall through an additional ordinary finite difference scheme. The numerical procedure is implemented in the commercial simulation software DSHplus. The capability of the code is examined by comparing the simulation results with theoretical solutions and experimental data.

KEYWORDS: Thermohydraulics, transient pipe flow, heat transfer, injection rate measurement, waterhammer

1. Introduction

Typically, the physical models used for the simulation of pipe flows with distinct transient features (e.g. waterhammer problems) only consider the continuity and momentum equation. If at all, changes in material properties are incorporated by an additional material law such as $\rho = \rho(p)$. Effects like heat transfer or transient changes of fluid properties (e.g. speed of sound, viscosity) due to temperature changes are typically not covered at all. In certain applications of thermohydraulics, these effects are vital. In order to model the heat transfer between fluid and pipe wall accurately in these cases, the temperature distribution in the pipe itself has to be considered, too. Based on these requirements, a physical model of the problem is developed.

2. Physical model

The problem is split into a liquid (pipe flow) and a solid (pipe wall) domain, for which separate (but coupled) physical models are used. The more complex physical model for the fluid domain is discussed first.

2.1. Fluid domain

The fluid behaviour is described by a set of three conservation equations in differential form. These differential equations are formulated in terms of primitive variables, since the simulation model should be capable of handling real fluids (and in particular, liquids), where analytical expressions for the state equations are not commonly available.

2.1.1. Continuity equation

The compressible continuity equation, expressed in terms of the area-averaged axial velocity $\bar{u}$ (volume flow Q by pipe cross-sectional area A) and fluid density ρ is given as follows:

$$\frac{\partial \rho}{\partial t} + \bar{u}\frac{\partial \rho}{\partial x} + \rho\frac{\partial \bar{u}}{\partial x} = \frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\frac{\partial \bar{u}}{\partial x} = 0 \tag{1}$$

Since primitive variables are used, the total density change has to be expressed in terms of the variables $\bar{u}$, p or T. Using the definition of the wave propagation speed a, the modified continuity equation can be expressed in terms of the fluid pressure p:

$$\frac{\partial \rho}{\partial t} + \bar{u}\frac{\partial \rho}{\partial x} + \rho\frac{\partial \bar{u}}{\partial x} = \frac{1}{a^2}\frac{\partial p}{\partial t} + \frac{\bar{u}}{a^2}\frac{\partial p}{\partial x} + \rho\frac{\partial \bar{u}}{\partial x} = 0 \tag{2}$$

2.1.2. Momentum equation

If the influence of gravity is neglected, the momentum equation (Navier-Stokes equation) in longitudinal pipe direction reads as follows:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\Delta p}{\Delta x} = 0 \tag{3}$$

The pressure loss per unit pipe length $\Delta p/\Delta x$ is calculated using the methodology described in section 2.1.4.

2.1.3. Energy equation

The energy equation, formulated in terms of the area-averaged thermodynamic fluid temperature T, can be written as follows /1/:

$$\rho c_p \frac{\mathrm{d}T}{\mathrm{d}t} - \frac{T}{v}\left(\frac{\partial v}{\partial T}\right)_p \frac{\mathrm{d}p}{\mathrm{d}t} - \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) - \frac{4}{d}\dot{q}'' - \left|\bar{u}\frac{\Delta p}{\Delta x}\right| = 0 \tag{4}$$

In this equation, k denotes the thermal conductivity of the fluid. The prefactor of the pressure differential $\mathrm{d}p/\mathrm{d}t$ is the product of the fluid temperature and the isobaric thermal expansion coefficient:

$$\frac{T}{v}\left(\frac{\partial v}{\partial T}\right)_p = T\gamma_p \tag{5}$$

For an ideal gas, the isobaric thermal expansion coefficient is equal to $1/T$, rendering the prefactor equal to one. For hydraulic oils and other liquids, γ_p is estimated numerically from the state function $v = v(p,\ T)$ provided by the DSHplus fluid library. Typical values for the prefactor are given in **table 1**. The values were calculated for ambient pressure (p = 1 bar) by using a central difference approximation at the respective temperature with a step size of $\Delta T = \pm$ 5 K.

Fluid	$T\gamma_p(T$ = 20 °C)	$T\gamma_p(T$ = 100 °C)
Water	0.06	0.29
HLP 46	0.21	0.29
Diesel	0.24	0.33
Gasoline	0.32	0.45

Table 1: Product of fluid temperature and expansion coefficient for various fluids.

For the ease of calculation when using the Method of Characteristics (as will be described later), the pressure differential $\mathrm{d}p/\mathrm{d}t$ in the energy equation has to be replaced by a partial derivative. By using the modified continuity equation (2), it can be expressed as a function of the spatial derivative of the flow velocity:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\rho a^2 \frac{\partial \bar{u}}{\partial x} \tag{6}$$

The wave propagation speed a differs from the fluid's speed of sound a_0, since every pipe is made of a material with finite Young's modulus E. The relations for estimating the wave propagation speed for different mechanical support conditions of the pipe are taken from a literature overview provided by HALLIWELL /2/. For linear-elastic pipe material, the following quantities are needed to calculate the wave propagation speed:

$$a = a\left(a_0, \rho, E, \mu, \frac{D}{d}\right) \tag{7}$$

2.1.4. Pressure loss

For laminar pipe flows, ZIELKE /3/ demonstrated that the pressure loss Δp can be decomposed into steady and unsteady components. The pressure loss due to steady flow is calculated using the well-known Darcy-Weisbach equation with the Darcy friction factor λ. The friction factor can be obtained as a function of the pipe diameter Reynolds number Re_d from theoretical considerations (laminar case) or experimental results. The pressure loss also depends on the relative roughness ε/d of the inner pipe surface, if the roughness ε exceeds the viscous sublayer thickness in turbulent flow.

In contrast to steady flow conditions, the estimation of pressure losses due to unsteady flow is much more demanding. For the laminar case, a classical approach to the problem has been presented by ZIELKE /3/. For better numerical calculation performance, the approximation of the weighting function presented by MÜLLER /4/ is used. For turbulent flow through smooth and rough pipes, VARDY & BROWN /5/ have published numerical schemes that will be implemented in the near future.

2.1.5. Heat transfer

The area-specific heat flux $\dot{q}''$ between fluid and pipe wall is calculated using the inner heat transfer coefficient α_d, the wall temperature T_w and the mean fluid temperature T:

$$\dot{q}'' = \alpha_d (T_w - T) \tag{8}$$

As is the case with the friction factor λ, the heat-transfer coefficient is calculated from analytical solutions for selected special cases or – in general – experimental correlations. Typically, the heat transfer coefficient is represented in a non-dimensional form, namely the Nußelt number Nu_d, in the literature:

$$Nu_d = \alpha_d \frac{d}{k} \tag{9}$$

In general, the Nußelt number Nu_d for a given problem depends on the geometry, the Reynolds number Re_d, the Prandtl number Pr of the fluid (ratio of momentum diffusivity to thermal diffusivity), the relative length L/d of the pipe section (if the velocity profile is not fully developed) and also on the relative surface roughness ε/d:

$$Nu_d = Nu_d \left(Re_d, Pr, \frac{L}{d}, \frac{\varepsilon}{d} \right) \tag{10}$$

Because the area-specific heat flux also depends on the pipe wall temperature, the equations of both domains are coupled.

2.2. Wall domain

Since the pipe wall is a solid body and its elastic deformations are either not considered (axial) or taken into account by the equations of the fluid domain (radial), the only remaining conservation equation for the solid domain is the energy equation. Its single variable (other than the fluid temperature) is the wall temperature T_w, which is a function of time and axial position along the pipe. Since the pipe material is of finite thickness, T_w may also vary in radial direction. However, the following considerations will prove that for most applications, the radial temperature profile can be neglected.

2.2.1. Radial temperature distribution

In the current approach, the significance of the radial temperature distribution within the wall domain is characterised by the Biot number Bi. The Biot number is defined as the ratio of the conductive thermal resistance R_{cond} of a body to the convective thermal resistance R_{conv} at the surface of the body. If the thermal resistance between the fluid and the pipe wall is used as the reference convective resistance, it can be shown that the Biot number equals the following temperature ratio:

$$Bi = \frac{R_{cond}}{R_{conv}} = \frac{T_W\left(\frac{d}{2}\right) - T_W\left(\frac{D}{2}\right)}{T - T_W\left(\frac{d}{2}\right)} \tag{11}$$

If the Biot number is much less than one, the radial distribution of the wall temperature is negligible. Using the definition of the Nußelt number, the following relation is derived for the Biot number:

$$Bi = \frac{\alpha_d d}{2k_w} \ln\left(\frac{D}{d}\right) = \frac{Nu_d}{2} \frac{k}{k_w} \ln\left(\frac{D}{d}\right) \tag{12}$$

Numerical analysis of equation (12) shows that for many thermohydraulic and almost all pneumatic problems, the Biot number is indeed much less than one (for Diesel flowing through a steel pipe with D/d = 1.5 at Re_d = 10^4, $Bi \approx 0.08$). Hence, the radial distribution of the wall temperature can be neglected for these applications.

2.2.2. Energy equation

If this major simplification is applied and radiative heat transfer (which is easily taken into account by linearization of the biquadratic temperature terms, if required) is neglected, the energy equation for an infinitesimal slice of the pipe wall reads as follows:

$$\rho_w c_w \frac{\partial T_w}{\partial t} = k_w \frac{\partial^2 T_w}{\partial x^2} + \frac{4}{(D^2 - d^2)}\left[\alpha_d d(T - T_w) + \alpha_D D(T_u - T_w)\right] + \frac{4D}{(D^2 - d^2)} \dot{q}_e'' \tag{13}$$

In this equation, $\dot{q}_e''$ denotes the area-specific power of possible external heating devices, while T_u equals the temperature of the fluid surrounding the pipe wall.

3. Solution methods

In general, the balance equations derived in the previous chapters cannot be solved analytically. Hence, numerical methods are employed to solve the set of equations approximately. The equations governing the fluid and wall domain are solved simultaneously. For solving the equations describing the fluid behaviour, the Method of Characteristics is used. A standard finite difference scheme is employed for solving the energy equation of the wall domain.

3.1. Method of Characteristics (fluid domain)

The Method of Characteristics (MOC) is a very efficient scheme for solving systems of partial differential equations numerically. However, one of the major drawbacks is its limitation to hyperbolic equations. Since a diffusive term ($k\,\partial^2 T/\partial x^2$, if thermal conductivity k is assumed constant within a timestep) is present in the energy equation, the governing set of equations is not strictly hyperbolic. The diffusive term, representing Fourier's law, is used to model the axial heat transfer due to thermal conduction within the fluid. The only other mode of heat transfer in axial direction (within the fluid domain) is due to convective transport of fluid elements and their respective energies. The relative significance of axial convective heat transfer compared to axial thermal conduction can be characterized by the Péclet number $Pé$. The Péclet number, written in terms of pipe flow problems, is defined as follows:

$$P\acute{e} = \frac{\bar{u}\rho c_p d}{k} \tag{14}$$

For typical thermohydraulic problems, the Péclet number is much greater than one. Hence, the axial heat transfer in the fluid domain is governed by convection and the contribution of thermal conduction (and therefore, the diffusive term in the energy equation) can be neglected. The validity of this statement will be proven by comparison with analytical solutions in section 4.1.

After applying this simplification, the set of equations becomes hyperbolic, and the MOC can be employed. By performing a suitable manipulation, the set of partial differential equations is transformed into a set of ordinary differential equations (the so-called standard form). A procedure for obtaining the standard form for this type of equations is discussed in detail by PEUSSNER /6/. For the presented set of equations, the standard form reads:

$$\frac{\mathrm{d}p}{\mathrm{d}t} + \rho a \frac{\mathrm{d}\bar{u}}{\mathrm{d}t} + a\frac{\Delta p}{\Delta x} = 0 \text{ for } \frac{\mathrm{d}x}{\mathrm{d}t} = \bar{u} + a \quad (15)$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} - \rho a \frac{\mathrm{d}\bar{u}}{\mathrm{d}t} - a\frac{\Delta p}{\Delta x} = 0 \text{ for } \frac{\mathrm{d}x}{\mathrm{d}t} = \bar{u} - a \quad (16)$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} - \frac{\rho c_p}{T\gamma_p}\frac{\mathrm{d}T}{\mathrm{d}t} + \frac{1}{T\gamma_p}\left(\frac{4}{d}\dot{q}'' + \left|\bar{u}\frac{\Delta p}{\Delta x}\right|\right) = 0 \text{ for } \frac{\mathrm{d}x}{\mathrm{d}t} = \bar{u} \quad (17)$$

The total derivatives in these three equations are replaced by first-order finite differences. Since the differential equations are only ordinary along certain curves ("Characteristics") in the time-space grid diagram (**Figure 1**), a total derivative of the same physical quantity has to be approximated by different finite difference expressions, depending on the respective equation and propagation speed $\mathrm{d}x/\mathrm{d}t$:

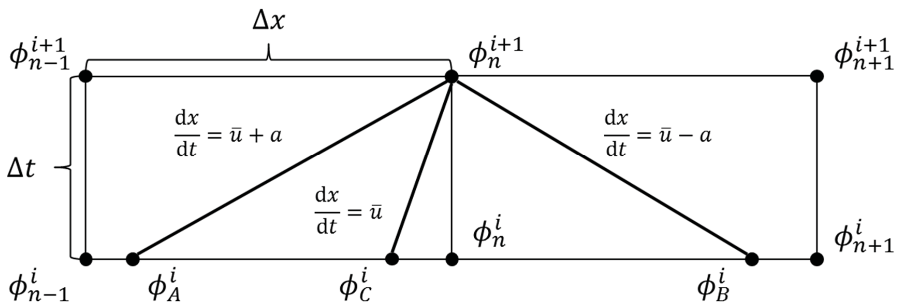

Figure 1: Time-space grid diagram and characteristic speeds.

For approximating the total derivative of an arbitrary quantity ϕ occurring in equation (15), where changes propagate with the speed $\mathrm{d}x/\mathrm{d}t = \bar{u} + a$, the value of ϕ at node 'A' and time step i has to be used when formulating the finite difference:

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} \approx \frac{\Delta\phi}{\Delta t} = \frac{\phi_n^{i+1} - \phi_A^i}{\Delta t} \quad (18)$$

For transforming equations (16) and (17) into algebraic expressions, the values at the nodes 'B' and 'C' respectively, have to be used. The values of ϕ at nodes 'A', 'B' and 'C' are determined by linear interpolation between the adjacent nodes. After all derivatives of the flow variables are replaced according to this scheme, the resulting set of algebraic equations can be solved with respect to the values at the node n and the time $i + 1$. Once the fluid variables $\bar{u}$, p and T at the time step $i + 1$ have been calculated, the associated new wall temperature can be computed.

3.2. Finite difference method (wall domain)

Since conduction is the only mechanism by which the pipe wall can transfer heat in its axial direction, the diffusive expression in the energy equation cannot be neglected.

Hence, the (classical) Method of Characteristics cannot be used for solving the non-hyperbolic energy equation governing the wall domain. Instead, a standard finite difference method is used. The time derivative of the wall temperature is approximated by a first-order forward scheme, whereas the second order spatial derivative of the wall temperature (Fourier's law) is calculated by a central difference scheme. Once the partial derivatives in the energy equation of the wall (equation 13) have been replaced by these expressions, the resulting algebraic equation can be solved for the wall temperature at the time step $i + 1$. After the new wall temperature is calculated, the whole procedure is repeated by solving the fluid equations based on this result.

3.3. Stability conditions

The proposed numerical scheme is implemented using an equally spaced grid in both time and space. Since the simulation time step size Δt is typically chosen with respect to the dynamics of the system to be simulated, the spatial step size Δx has to be determined based on stability conditions.

When applying the MOC, care has to be taken that a disturbance (e.g. pressure wave) cannot propagate farther than Δx in a given time increment Δt. This condition is met if the so-called Courant-Friedrichs-Lewy Number CFL is less than or equal to one:

$$CFL = \max\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)\frac{\Delta t}{\Delta x} = (|\bar{u}_{max}| + |a_{max}|)\frac{\Delta t}{\Delta x} \leq 1 \tag{19}$$

The finite difference scheme for solving the diffusive energy equation describing the pipe wall is stable if the following condition is satisfied:

$$\frac{k_w}{\rho_w c_w}\frac{\Delta t}{\Delta x^2} \leq \frac{1}{2} \tag{20}$$

Comparison between the two stability conditions shows that for typical fluid power systems, equation (19) imposes stricter conditions on the time step size and spatial discretization than equation (20). Hence, the stability condition of the fluid domain is used for determining the spatial step size Δx.

4. Examples

The numerical scheme is implemented as a new class of pipe models into the commercial fluid power simulation software DSHplus. The capabilities of the new pipe model are demonstrated by comparing simulation results with analytical solutions for a simplified case (constant wall temperature) and experimental data.

4.1. Axial temperature distribution in a fluid, constant wall temperature

If we assume the wall temperature, mean flow velocity, inner heat transfer coefficient and all fluid parameters to be constant, an analytical solution for the axial temperature distribution of the fluid can be given. If the influence of the pressure change on the fluid's enthalpy is neglected, the energy balance for a fluid element (with the cross-sectional area A) leads to the following ordinary differential equation:

$$kA\frac{\mathrm{d}^2T}{\mathrm{d}x^2} - \bar{u}A\rho c_p\frac{\mathrm{d}T}{\mathrm{d}x} + \alpha_d\pi d(T_w - T) + A\bar{u}\frac{\Delta p}{\Delta x} = 0 \quad (21)$$

To obtain a homogeneous representation of the energy balance, the excess temperature Θ is introduced:

$$\theta = T - T_w - \frac{\Delta p}{\Delta x}\frac{\bar{u}d}{4\alpha_d} \quad (22)$$

If we denote the fluid temperature at the starting point of the heated pipe section with $T(x = 0) = T_0$ and demand the fluid temperature to attain a constant value for $x \to \infty$, the axial distribution of the excess temperature $\Theta = \Theta(x)$ is given by the following expression:

$$\theta(x) = \left(T_0 - T_w - \frac{\Delta p}{\Delta x}\frac{\bar{u}d}{4\alpha_d}\right)e^{\frac{x}{2d}\left[Pé - \sqrt{Pé^2 + 16\frac{\alpha_d Pé}{\bar{u}\rho c_p}}\right]} = \theta_0 e^{\frac{x}{2d}\left[Pé - \sqrt{Pé^2 + 16\frac{\alpha_d Pé}{\bar{u}\rho c_p}}\right]} \quad (23)$$

Figure 2 shows the comparison between the simulated axial temperature distribution ($Pé = \infty$) and the analytical solution for Péclet numbers ranging from $0.02 < Pé < \infty$, represented by the following non-dimensional quantities:

$$\theta^*(x) = \frac{\theta(x)}{\theta_0}, x^* = 4\frac{x}{d}\frac{\alpha_d}{\bar{u}\rho c_p} \quad (24)$$

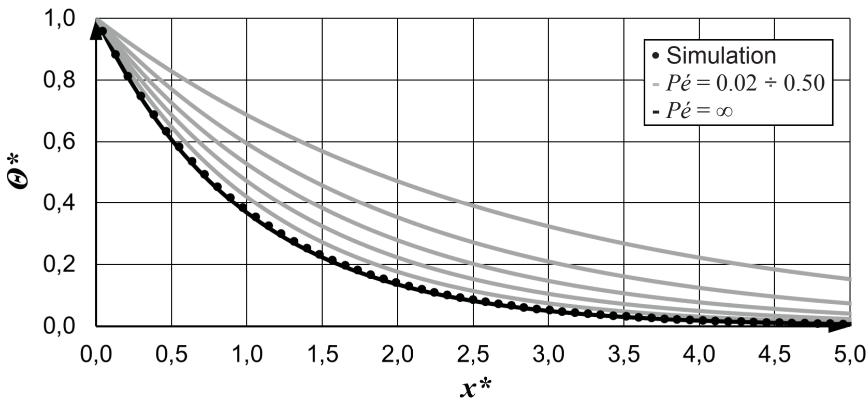

Figure 2: Axial fluid temperature distribution for constant wall temperature.

Apparently, the numerical simulation shows excellent agreement with the analytical solution for the case of negligible axial heat conduction.

4.2. Injection rate measurement

The shape of the fuel injection rate (mass or volume flow vs. time) has significant effects on the efficacy and emissions of internal combustion engines /7/. To analyse the injection rate of a given injection system experimentally, specialized test benches are used. Assuming constant fluid properties, the pressure signal $p(t)$ measured in a pipeline following the injector can be transformed into the corresponding volume flow rate $Q(t)$, if the pipe impedance is known. However, this assumption is not always satisfied, in particular if the fluid properties (and the pipe impedance) change along the pipeline due to heat transfer as is the case for an existing test bench /8/. In addition, the duration of injection is of the order of a millisecond (transient effects), which makes this scenario an ideal test case for the developed numerical model.

The experimental set-up features an injector, for which the volume flow rate distribution $Q(t)$ is approximately known, and a sufficiently long pipe, along which significant heat transfer ($\Delta T/\Delta x \approx$ -12 K/m) occurs. A pressure sensor measures the resulting pressure distribution $p(t)$ in the pipe section. For comparison, both diabatic and isothermal conditions are studied. To maintain isothermal conditions, heating devices along the pipeline are used to prevent temperature loss. To exclude reflections of pressure waves that could alter the pressure signal, a non-reflecting line termination (“RaLa”) is used. The measured pressure distributions, along with the corresponding numerical results, are presented in the following diagram (**Figure 3**):

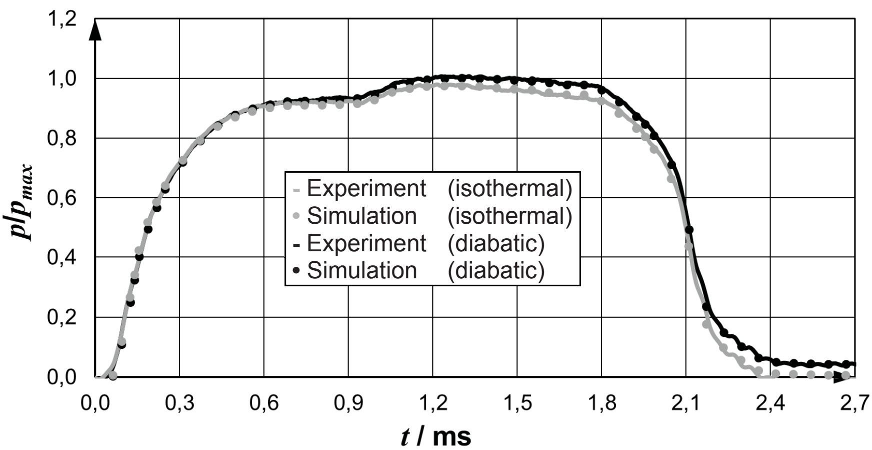

Figure 3: Pressure distribution for varying thermal boundary conditions /8/.

As can be seen, the heat transfer along the pipe leads to a significant pressure rise due to changes in the hydraulic impedance of the pipeline, which underlines the importance of thermohydraulic effects. The numerical results for both isothermal and diabatic flow conditions show excellent agreement with the respective experimental data.

5. Discussion

A numerical model to simulate transient pipe flows with heat transfer was developed and successfully implemented into the simulation software DSHplus. Comparison with an analytical solution for a simplified case and selected experimental data of transient diabatic pipe flow shows excellent agreement with the numerical model. Future developments will see the implementation of procedures to cover the effects of unsteady turbulent pipe friction and unsteady heat transfer coefficients. The developed model also allows the analysis of the influence of heat transfer on the hydraulic transmission characteristics (characterized by a four-pole matrix) of a pipe section, which will be covered in an additional paper.

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7. Nomenclature

a	Wave propagation speed	m/s
a_0	Speed of sound in the fluid	m/s
A	Cross-sectional area of the pipe	m^2
Bi	Biot number	1
c_p	Specific isobaric heat capacity of the fluid	$J \cdot kg^{-1} \cdot K^{-1}$
c_w	Specific heat capacity of the wall	$J \cdot kg^{-1} \cdot K^{-1}$

d	Inner pipe diameter	m
D	Outer pipe diameter	m
k	Heat conductivity of the fluid	$W \cdot m^{-1} \cdot K^{-1}$
k_w	Heat conductivity of the wall	$W \cdot m^{-1} \cdot K^{-1}$
Nu_d	Nußelt number, inner wall surface	1
p	Thermodynamic pressure	N/m^2
Pr	Prandtl number of the fluid	1
$\dot{q}_e''$	Area-specific power of external heat sources	$W \cdot m^{-2}$
Re_d	Pipe flow Reynolds number	1
T	Area averaged fluid temperature	K
T_w	Wall temperature	K
$\bar{u}$	Area averaged fluid velocity	m/s
v	Specific volume	$m^3 \cdot kg^{-1}$
α_d	Inner heat transfer coefficient	$W \cdot m^{-2} \cdot K^{-1}$
α_D	Outer heat transfer coefficient	$W \cdot m^{-2} \cdot K^{-1}$
γ_p	Isobaric thermal expansion coefficient	K^{-1}
ε	Relative roughness	m
θ	Excess temperature of the fluid	K
λ	Darcy friction factor	1
μ	Poisson's ratio of the pipe material	1
ρ	Fluid density	$kg \cdot m^{-3}$
ρ_w	Density of the pipe material	$kg \cdot m^{-3}$